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Limitation of Operational Definitions

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Fundamental ideas underlying the definition of the properties of physical systems by means of measurement procedures in the paradigm of operationalism are introduced and discussed. Minimal criteria that such procedures might be expected to satisfy are suggested and a fundamental limitation to operationalism is pointed out.

1. INTRODUCTION

Bridgman (1927) presents a lucid discussion of the merits of the operational definition of physical quantities, being careful to also point out certain difficulties associated with such an approach. Among the many important ideas he presents, the following are especially relevant for the purposes of this paper:

1. The fundamental importance of (measurement) operations in defining physical quantities is elucidated in the statement "*the concept is synonymous with the corresponding set of operations*" (author's italics).

An advantage of operationalism is its capacity to clarify the nature of physical quantities. For example, various quantities commonly referred to as "properties" of objects are revealed to actually be relations between the object and its observer (such distinctions being of crucial importance in certain relativistic and quantum mechanical contexts).

2. The problem of defining such operation sets is discussed in general terms for several fundamental quantities (distance, time, energy, etc.). Significant difficulties are evidenced, especially in the discussion concerning the definition of fundamental quantities like distance and time at different (microscopic, "everyday," and astronomical) spatial and temporal scales. A particu-

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larly incisive remark is made about the nonexistence of a criterion for determining if a given object is a clock (Bridgman, 1927, p. 72).

3. The above-mentioned operation sets are characterized as being necessarily unique, implying that quantities defined by different such sets would have to be understood to be different and consequently they would have to be denoted by different names (possible examples being fundamental quantities like distance or time at different scales). The operations themselves are characterized as necessarily being effectively executable. The possibility that some operations might be "mental" ones is not excluded.

The operational approach to defining properties as characterized in the above is commented on in Section 2, and some possibly significant properties of the above-mentioned operation sets are suggested. Section 3 presents a more formal discussion of operationalism, including a demonstration of an apparent limitation of that approach. Section 4 contains concluding remarks.

2. CHARACTERISTICS OF OPERATIONAL DEFINITIONS

With reference to point 1 above, it seems to the present author that the term "operation set" is best interpreted as "operation sequence," that is, "procedure". Indeed this interpretation seems obvious in various parts of Bridgman (1927) and the word "procedure" is even used. In any case, "procedure" is the interpretation assumed in the following, and frequent reference will be made to "measurement procedures." Such procedures are understood to be analogous to those used for computers, in that they specify operations to be performed and the order in which they must be performed. However, the operations are not limited to be computational ones; they can be any operation that a physical system can perform. It is understood that such a procedure operates on physical systems and is executed by a physical system. Indeed, a reference to a measurement procedure in the following should be considered to be also a reference to the system executing it. Reference is typically made to the procedure instead of to its executor because the properties under discussion are typically those which the procedure has independent of which system executes it.

With reference to the difficulties alluded to in point 2 above, it is remarkable that still no procedures seem to exist for the operational definition of properties. Perhaps this is just due to the difficulty of the problem. If this is the case one might consider various options before giving up on such an important and interesting idea as operationalism. One approach might be to try to define procedures for quantities which are "simpler" than distance, time, etc.

However, before actually trying to realize measurement procedures it might be useful to first try to better understand the problem by defining general properties of such procedures. In point 3 above the properties of uniqueness and effective executability were mentioned. It is not difficult to specify various other properties that seem logically necessary, such as the following.

- *Completeness*, which refers to the capacity of a measurement procedure to define a property using the same procedure over a whole range of conditions. The difficulty of defining properties under conditions spanning different spatial and temporal scales can be recognized as a difficulty in realizing the relevant measurement procedure in such a way that it exhibits an adequate level of completeness.
- *Finiteness*, which refers to the capacity of a measurement procedure to produce a result (property value) in a finite time. Such a requirement would seem to exclude the possibility of procedures that measure infinite magnitudes and ones that can repeat operations indefinitely.

Considering the remark about "mental" operations in point 3 above, an operation that seems to have mental aspects is *computation* . Indeed some measurement procedures (especially those employed for defining properties at microscopic and astronomical scales) are characterized by a significant amount of computation, in contrast with some others (at the "everyday" level) that seem, at first sight, to be devoid of computational operations. As an example of the latter case Bridgman (1927) cites the use of a ruler to measure distance (as the number of times the ruler must be laid down end to end so as to cover the distance). It must be pointed out, however, that counting itself can be viewed as a computational operation, so computation may well be much more characteristic of measurement than is apparent at first sight.

3. MEASUREMENT PROCEDURE CRITERIA AND LIMITATIONS

This section presents a more formal treatment of measurement procedures and the properties they define in the context of the paradigm of operationalism as characterized in the preceding.

For greater clarity and preciseness it is convenient to employ terminology analogous to that used to define mathematical functions. Thus, a measurement procedure is characterized by a triple (DOMAIN, CODOMAIN, FUNCTION) where

DOMAIN is its domain of applicability
CODOMAIN is its codomain (the values it can produce)
FUNCTION means what it does

In line with the operational point of view being followed, it is assumed that the domain of measurement procedures is the set of all physical systems (PHYSICAL-SYSTEMS), i.e.,

$$\text{DOMAIN} = \text{PHYSICAL-SYSTEMS}$$

In abstract terms the FUNCTION of a measurement procedure is to determine property values for physical systems, and thereby to define the property itself. It is assumed that a valid definition of a property by means of a measurement procedure requires that the measurement procedure satisfy the following criterion: A measurement procedure always yields a value for a system having the property defined by the procedure and never yields a value for a system which does not have that property.

So characterized, a measurement procedure individuates a subset of PHYSICAL-SYSTEMS, and the different possible values in its CODOMAIN determine a partition of that subset into equivalence classes.

The yielded property value must be produced in a finite time. Precisely, "yielding a property value" is understood to include two distinct aspects: determining the value and "returning" the value, this latter term meaning the outputting of information identifying the value as the measurement result in an unambiguous and clearly recognizable form (in a finite time). If a procedure does not return a value under certain conditions, it will be said to "never return a value" (meaning never under those conditions). To take these considerations into account, the above-stated criteria that a measurement procedure must satisfy may be better expressed as:

- A measurement procedure determines a value and returns it for any system having the property defined by the procedure and never returns a value for any system which does not have that property.

The subset of systems for which the measurement procedure determines and returns a value corresponds to the property it defines.

The above criteria a measurement procedure must satisfy are not intended as a definition of the concept of "measurement procedure." Indeed such procedures are physical systems and, in the operational paradigm, their property of "being a measurement procedure" should be defined by a measurement procedure. The FUNCTION of the relevant procedure M can be described as follows: for any S in PHYSICAL-SYSTEMS, M determines whether it is a measurement procedure and returns

$$M(S) = 1 \quad \text{if } S \text{ is (i.e., executes) a measurement procedure (on systems comprising a nonempty subset of PHYSICAL–SYSTEMS)}$$

$$M(S) = 0 \quad \text{otherwise}$$

In words, the values in the CODOMAIN of M partition the set of physical systems into two classes, the class of physical systems that are measurement procedures [$M(S) = 1$] and the class of physical systems that are not measurement procedures [$M(S) = 0$].

Although nothing has been assumed about the internal workings of M, it has a serious problem which can be evidenced by studying another procedure, G, constructed using M, whose FUNCTION can be specified as follows: for a specified physical system S (any S in PHYSICAL-SYSTEMS)

$$\text{if } M(S) = 0, \qquad \text{return 0 as value of } G(S)$$

$$\text{if } M(S) = 1, \qquad \text{never return a value}$$

If one now considers $G(G)$, one concludes the following:

- $G(G)$ is a measurement procedure (it determines and returns a value) if, according to M, it is not a measurement procedure.
- $G(G)$ is not an measurement procedure (it does not determine and return a value) if, according to M, it is a measurement procedure.

This logical contradiction implies that G cannot exist, but since its existence depends only on the existence of M, then M cannot exist either. That is, no procedure can exist for the operational definition of the property of being a measurement procedure (the above demonstration is analogous to certain presentations of the proof of the nonexistence of an algorithm capable of deciding if an arbitrary Turing machine will halt given an arbitrary input).

4. CONCLUSIONS

The above result concerning the nonexistence of the procedure M contradicts the basic tenet of operationalism, i.e., that all properties of physical systems can be defined by means of measurement procedures (which are themselves physical systems).

It is important to realize that the above conclusions cannot be avoided by trying to formulate a denial that G is a physical system (on the basis of the way it is constructed). Obviously G has the same nature that M does, and M must be a physical system in the paradigm of operationalism.

Another way to try to avoid the above conclusions would be to reject the above criteria for being a measurement procedure. To reject the idea that such a procedure must determine a value seems absurd. To reject the idea that it must clearly identify what entity constitutes that value seems very dangerous, since that could open the door to various kinds of ambiguities. It should also be noted that the above demonstration depends on the criteria essentially only in the construction of G, which does rely on the assumption that M does determine and return a value.

Of course, the above demonstration does not deny the possibility of realizing measurement procedures for deciding if certain specific physical systems are measurement procedures. The problem is that no one procedure is sufficient and, from an operational point of view, multiple procedures imply multiple meanings (and thus *names* for “measurement procedure”).

REFERENCE

Bridgman, P. W. (1927). *The Logic of Modern Physics*, MacMillan, New York.